Mathematics at the rescue

Human being invented something very useful for everything is difficult to understand: mathematics.

Can you imagine the size of the earth? Hmmm it's hard. But if I tell you that the earth diameter is 12742km you can understand that. You know what is 1km and what time it takes to walk this far and you can extrapolate to this distance.
Nevertheless you never “experienced” such huge distance, just like you cannot experience quantum physics.

Mathematics **helps us** to perceive what is beyond our understanding. Thanks to mathematics we can understand the incomprehensible. And it will help us to apprehend quantum mechanics!

And don't be frightened with formulas, this is no big deal!

Probabilities

We saw with Schrodinger's cat that quantum behaviors are based on probabilities. At least this is the way we perceive it.
This chapter to remind you basics of this branch of mathematics (that I always hated to be honest).

Let's consider the following example: you flip a coin and get the result: heads or tails.
You have 50% chances to get heads and 50% chances to get tails.

The basic rule of probabilities is that the sum of events percentages must be 100%. But in formulas we don't use percentages, we use numbers called **probability amplitude**.

In the case of tossing a coin, we have two amplitudes:

- p_h to get heads,
- p_t to get tails.

With probabilities amplitude, the sum of squared absolute values of probability amplitudes must be 1 😎.

Said differently: $|p_h|^2+|p_t|^2=1$. You see? Mathematics helps!

Because we have the same chance to get heads or tails, $p_h=p_t=\frac{1}{\sqrt{2}}$. Let's check:

$$\left|\frac{1}{\sqrt{2}}\right|^2+\left|\frac{1}{\sqrt{2}}\right|^2=\frac{1}{2}+\frac{1}{2}=1$$

Imagine now you find a *trick* to change the chance to get heads or tails. Let's say 70% chances to get heads and 30% chances to get tails. We would have:

- $p_h = \sqrt{0.7} = 0.8366$
- $p_t = \sqrt{0.3} = 0.54772$

Let's check: $0.8366^2 + 0.54772^2 = 1$
Computed amplitudes are correct.

Superposition

Like Schrödinger's cat, quantum particles are in superposition between the generator and the detector. That is to say dead **and** alive.
The same for qubits: when superposed it is both |0> and |1>.
This state is described with the following equation:

$$|\psi > = \alpha\,|0 > + \beta\,|1 >$$

With:

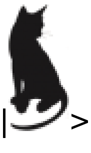
- $|\psi >$ the qubit state,
- α the amplitude of $|0 >$,
- β the amplitude of $|1 >$.

Between the generator and the detector the qubit is in a quantum state balanced by probabilities between |0> and |1>. When qubit hits the detector we read the result and get |0> or |1>.

If the generator is calibrated to launch particles with perfect balanced probabilities to get result |0> and |1>, here is the state of qubit between the source and the detector:

$$|\psi > = \frac{1}{\sqrt{2}}\,|0 > + \frac{1}{\sqrt{2}}\,|1 >$$

Which is, for the poor cat:

$|\psi > = \frac{1}{\sqrt{2}}$ |> $+ \frac{1}{\sqrt{2}}$ |>

Quantum matrices

I'll push a bit further with mathematics because there is another important way to describe quantum states: using matrices.

In the previous chapter we discovered the quantum state with the equation:

$$|\psi>=\alpha\,|0>+\beta\,|1>$$

This same state is also defined with the following matrix:

$$\begin{bmatrix}\alpha\\ \beta\end{bmatrix}$$

The top entry is the amplitude for |0> and the bottom entry the amplitude for |1>.

If you are not comfortable with matrices I invite you to review how it works and specifically how to multiply it. I won't do that here and you'll find plenty of courses about that on the web. For instance:

https://www.wikihow.com/Understand-the-Basics-of-Matrices

The matrix representing a qubit perfectly balanced between |0> and |1> is:

$$\begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix}$$

Previous: This little thing called qubit Next: Quantum states as matrices